Sequential Approach to QC Analysis of Inspection Process

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Confidential utility client

Some customers homes and business contaminated by mercury (Hg) release during improper removal of old equipment

Over 750,000 homes and businesses needed inspection and possible cleanup

Time frame—18 month period in early 2000’s
State Government Concerns

- State government was very skeptical
  - Utility’s ability to manage massive inspection project
  - Utility’s commitment and ability to ensure quality of inspections and remediation
- Anxious to have good news quickly
- Huge public health issue
- Legal action brought by the State Attorney General
  - civil penalties
  - direct state oversight of mercury inspection and cleanup program
Working with State Regulators

Key was gaining support of public health official on Task Force overseeing cleanup

Led to approving of Shaw’s management plan and settling of state’s legal action

Allowed the project to proceed expeditiously, without direct oversight

Based on innovative statistical approach
  – Binomial sequential probability ratio test (SPRT) statistic
  – Measuring and managing reliability of home inspection and testing procedures
What Shaw Provided

► Relief for client from civil penalties and direct state oversight of Hg inspection and cleanup program
► Management support, oversight of contractors and QA management
► Statistical support
► Billing approximately $10 million per week for much of project
SEQUENTIAL APPROACH TO QC ANALYSIS OF INSPECTION PROCESS

Overview of Problem
Top Level Objectives

- Demonstrate acceptable quality of inspection processes ASAP!
- Manage quality of inspection processes
  - Monitor
  - Report
  - Correct
  - Maintain
Inspection Processes

► Visual inspection
  – Certain factors evident from visual inspection could preclude presence of old type Hg regulators
  – Long checklist
  – Locations ‘Involved’ or ‘Not involved’ (NI)

► Hg inspection for ‘Involved’ locations
  – Jerome Hg vapor analyzer
  – ‘Instrument Clear’ (IC) or ‘Instrument Detect’ (ID)
Binomial Sequential Probability Ratio Test
Sequential Testing

- Observations taken in stages
- At each stage,
  - test statistic is computed and plotted
  - decision is made to reject or accept the hypothesis or to continue taking observations
- Each decision is made based on all the observations taken up to that point
- Process terminated by accepting or rejecting the statement when the value of the test statistic crosses a decision boundary
Hypotheses

For Visual Inspection, $p_V$ is the probability of error in visual inspection

$$H_{V_0} : p_V \leq p_{V_0} = 0.01 \text{ versus } H_{V_1} : p_V \geq p_{V_1} = 0.02$$

$p_J$ is the probability of error in Jerome Hg vapor analyzer inspection

$$H_{J_0} : p_J \leq p_{J_0} = 0.005 \text{ versus } H_{J_1} : p_J \geq p_{J_1} = 0.01$$
Set SPRT decision error rates ($\alpha$ and $\beta$) for both type of inspection to 5%

Decision boundaries are ($A < 1 < B$):

$$B \cong \frac{1-\alpha}{\beta}, \quad b = \ln B$$

$$A \cong \frac{\beta}{1-\alpha}, \quad a = \ln A$$

$$a < 0 < b$$

Nonparametric

Conservative
Distribution of independent trials with binary outcomes
- Number of Heads in coin tossing
- Number of correct answers on True/False test

Need
- Fixed number \((n)\) of trials
- Trials are independent of each other
- Probability of success \((p)\) is same from trial to trial

Formula:
\[
P\left(X = x \mid n, p\right) = \binom{n}{x} p^x (1 - p)^{n-x}, \quad 0 < p < 1
\]
For $0 < p_0 < p_1 < 1$, define the maximum joint likelihood under $H_0$: $p \leq p_0$ as

$$f_{m,0}^* = \max_{p \leq p_0} \left( \prod_{i=1}^{m} \binom{n_i}{x_i} p^{x_i} (1 - p)^{n_i - x_i} \right)$$

$$= \left[ \prod_{i=1}^{m} \binom{n_i}{x_i} \right] \hat{p}_{m,0}^{S_m} \bigg( 1 - \hat{p}_{m,0} \bigg)^{N_m - S_m},$$

$$\hat{p}_{m,0} = \min(\hat{p}_m, p_0),$$

$$\hat{p}_m = S_m / N_m, \quad S_m = \sum_{i=1}^{m} x_i, \text{ and } N_m = \sum_{i=1}^{m} n_i.$$
Similarly, define the maximum joint likelihood under $H_1: p \geq p_1$ as

$$f_{m,1}^* = \max_{p \geq p_1} \left( \prod_{i=1}^{m} \binom{n_i}{x_i} p^{x_i} (1-p)^{n_i-x_i} \right)$$

$$= \left[ \prod_{i=1}^{m} \binom{n_i}{x_i} \right] \hat{p}_{m,1}^{s_m} \left( 1 - \hat{p}_{m,1} \right)^{n_m - s_m},$$

$$\hat{p}_{m,1} = \min(\hat{p}_m, p_1).$$
Probability Ratio for sequence of binomial observations

\[
L_m = \frac{f_{m1}^*}{f_{m0}^*} = \frac{\hat{p}_{m1}}{\hat{p}_{m0}} \frac{S_m (1-\hat{p}_{m1})^{N_m-S_m}}{S_m (1-\hat{p}_{m0})^{N_m-S_m}} = \begin{cases} 
\left( \frac{p_1}{\hat{p}_m} \right)^{S_m} \left( \frac{1-p_1}{1-\hat{p}_m} \right)^{N_m-S_m}, & \text{for } \hat{p}_m \leq p_0 \\
\left( \frac{p_1}{p_0} \right)^{S_m} \left( \frac{1-p_1}{1-p_0} \right)^{N_m-S_m}, & \text{for } p_0 < \hat{p}_m < p_1 \\
\left( \frac{\hat{p}_m}{p_0} \right)^{S_m} \left( \frac{1-\hat{p}_m}{1-p_0} \right)^{N_m-S_m}, & \text{for } p_1 \leq \hat{p}_m
\end{cases}
\]
Sequential Probability Ratio Test

\[ Z_m = \ln L_m = \left( C_{1,m} + C_{2,m} \right) S_m - C_{2,m} N_m, \text{ where} \]

\[ C_{1,m} = \ln \left( \frac{r_{1,m}}{r_{0,m}} \right), \quad C_{2,m} = \ln \left( \frac{1 - r_{1,m}}{1 - r_{0,m}} \right) \]

- When \( H_0 \) true, \( Z_m \to -\infty \)
- When \( H_1 \) true, \( Z_m \to +\infty \)
- Use Wald’s decision boundaries \( a \) and \( b \)
  - When \( Z_m < a \), reject \( H_1 \); accept \( H_0 \)
  - When \( Z_m > b \), reject \( H_0 \); accept \( H_1 \)
  - When \( a < Z_m < b \), continue sampling
From previous slide, we see $Z_m$ is
- a function of the cumulative sum $S_m$
- a random walk with
  - Drift
    $$E Z_m = N_m \left[ p C_{1,m} + (1 - p) C_{2,m} \right]$$
  - Increasing variance
    $$\text{var}(Z_m) = N_m p (1 - p) \left( C_{1,m} + C_{2,m} \right)$$
- asymptotically Gaussian, when suitably normalized
- a martingale for $p = C_2/(C_1 + C_2)$
- a sub-martingale for $p > C_2/(C_1 + C_2)$
- a super-martingale for $p < C_2/(C_1 + C_2)$
A practical solution
QA Surveillance Strategy

- Normal surveillance rate—1 QA check per inspector per day
- Enhanced surveillance rate—2 QA checks per inspector per day
- Start under normal surveillance
- Single detected clearance error puts inspector under enhanced surveillance for the next 5 days
- Another detected clearance error renders him ineligible to conduct inspections
- All work performed by disqualified inspector must be checked
Test whether false clearance error rates acceptably low

Test statistic $Z_m$ used as quality indicator

Observations not terminated until all inspections complete

Wald’s lower boundary $a$ is used but does not terminate sampling

Test statistic passing $a$ at any time indicates clearance error rates are acceptably low
Three consecutive points moving closer to fail decision boundary triggers corrective action investigation.

Test statistic passing fail boundary \((b)\) is strong indication of need for corrective action, including:

- rechecking residences,
- retraining inspectors
- releasing inspectors
- firing subcontractor
## Some Early Stage Statistics (3/1/2001)

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<th>Symbol</th>
<th>Value</th>
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<td>Num NI checked</td>
<td>Nn</td>
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<tr>
<td>Num NI found ID</td>
<td>Xn</td>
<td>0</td>
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<tr>
<td>Rate of NI ID</td>
<td>Est. pn</td>
<td>0.00%</td>
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<td>Num Involved checked</td>
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<td>Num Involved found IC</td>
<td>Ni - Xi</td>
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<tr>
<td>Rate of Involved ID</td>
<td>Est. pi</td>
<td>0.80%</td>
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Statistical Quality Support

- Developed statistical methodology
- Won over key regulator
- Developed and installed software for
  - Client/server data entry and management system
  - Statistical analysis
  - Automated reporting (run and emailed nightly)
- Trained project personnel on site
- Operational
  - DBA
  - Software maintenance
  - Monitoring data entry and report generation
- Statistical consulting—error surveys
References
