Sequential Approach to QC Analysis of Inspection Process

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Project for Confidential Client

- Confidential utility client
- Some customers homes and business contaminated by mercury (Hg) release during improper removal of old equipment
- Over 750,000 homes and businesses needed inspection and possible cleanup
- Time frame—18 month period in early 2000’s

State Government Concerns

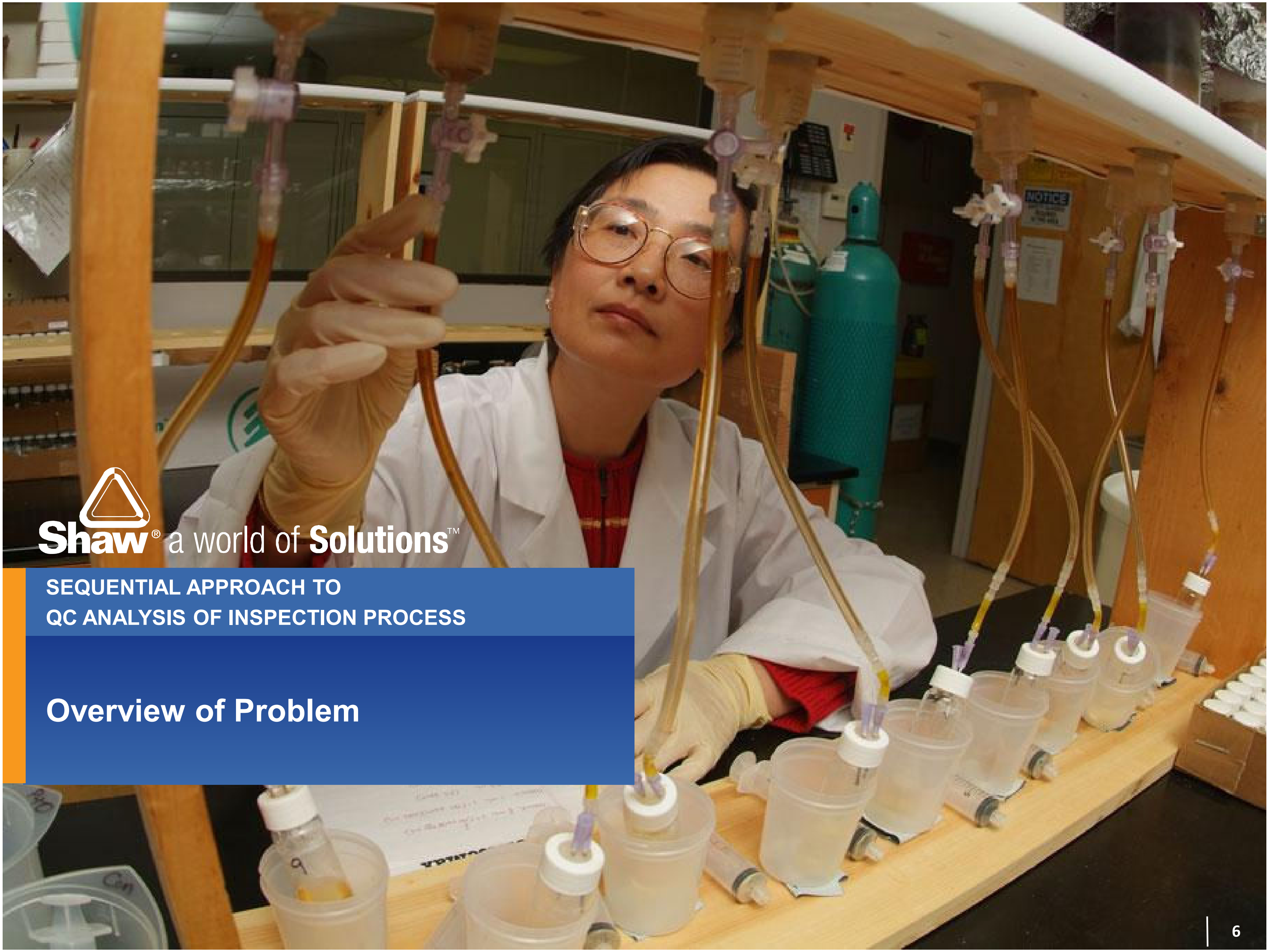
- State government was very skeptical
  - Utility's ability to manage massive inspection project
  - Utility's commitment and ability to ensure quality of inspections and remediation
- Anxious to have good news quickly
- Huge public health issue
- Legal action brought by the State Attorney General
  - civil penalties
  - direct state oversight of mercury inspection and cleanup program

Working with State Regulators

- Key was gaining support of public health official on Task Force overseeing cleanup
- Led to approving of Shaw's management plan and settling of state's legal action
- Allowed the project to proceed expeditiously, without direct oversight
- Based on innovative statistical approach
  - Binomial sequential probability ratio test (SPRT) statistic
  - Measuring and managing reliability of home inspection and testing procedures

What Shaw Provided

- Relief for client from civil penalties and direct state oversight of Hg inspection and cleanup program
- Management support, oversight of contractors and QA management
- Statistical support
- Billing approximately $10 million per week for much of project



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SEQUENTIAL APPROACH TO
QC ANALYSIS OF INSPECTION PROCESS

Overview of Problem

Top Level Objectives

- Demonstrate acceptable quality of inspection processes ASAP!
- Manage quality of inspection processes
  - Monitor
  - Report
  - Correct
  - Maintain

Inspection Processes

- Visual inspection
  - Certain factors evident from visual inspection could preclude presence of old type Hg regulators
  - Long checklist
  - Locations ‘Involved’ or ‘Not involved’ (NI)
- Hg inspection for ‘Involved’ locations
  - Jerome Hg vapor analyzer
  - ‘Instrument Clear’ (IC) or ‘Instrument Detect’ (ID)

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Binomial Sequential Probability Ratio Test

Sequential Testing

- Observations taken in stages
- At each stage,
  - test statistic is computed and plotted
  - decision is made to reject or accept the hypothesis or to continue taking observations
- Each decision is made based on all the observations taken up to that point
- Process terminated by accepting or rejecting the statement when the value of the test statistic crosses a decision boundary

Hypotheses

- For Visual Inspection, p_V is the probability of error in visual inspection

$$H_{V0}: p_V \le p_{V0} = 0.01 \text{ versus}$$
$$H_{V1}: p_V \ge p_{V1} = 0.02$$

- p_J is the probability of error in Jerome Hg vapor analyzer inspection

$$H_{J0}: p_J \le p_{J0} = 0.005 \text{ versus}$$
$$H_{J1}: p_J \ge p_{J1} = 0.01$$

SPRT Decision Boundaries

- Set SPRT decision error rates (α and β) for both type of inspection to 5%
- Decision boundaries are (A < 1 < B):

$$B \cong \frac{1-\alpha}{\beta}, \; b = \ln B$$

$$A \cong \frac{\beta}{1-\alpha}, \; a = \ln A$$

$$a < 0 < b$$

- Nonparametric
- Conservative

Binomial Distribution

- Distribution of independent trials with binary outcomes
  - Number of Heads in coin tossing
  - Number of correct answers on True/False test
- Need
  - Fixed number (*n*) of trials
  - Trials are independent of each other
  - Probability of success *(p)* is same from trial to trial
- Formula:

$$P\left(X = x \middle| n, p\right) = \binom{n}{x} p^x \left(1 - p\right)^{n-x}, 0 < p < 1$$

Maximized Binomial Likelihoods

- For $0 < p_0 < p_1 < 1$, define the maximum joint likelihood under H_0: $p \le p_0$ as

$$f_{m,0}^{*} = \max_{p \le p_0} \left(\prod_{i=1}^{m} \binom{n_i}{x_i} p^{x_i} (1-p)^{n_i - x_i} \right)$$

$$= \left[\prod_{i=1}^{m} \binom{n_i}{x_i} \right] \hat{p}_{m,0}{}^{S_m} \left(1 - \hat{p}_{m,0}\right)^{N_m - S_m},$$

$$\hat{p}_{m,0} = \min\left(\hat{p}_m, p_0\right),$$

$$\hat{p}_m = S_m / N_m, \; S_m = \sum_{i=1}^{m} x_i, \text{ and } N_m = \sum_{i=1}^{m} n_i$$

Maximized Binomial Likelihoods

- Similarly, define the maximum joint likelihood under H_1: $p \geq p_1$ as

$$f_{m,1}^{*} = \max_{p \geq p_1} \left(\prod_{i=1}^{m} \binom{n_i}{x_i} p^{x_i} \left(1 - p\right)^{n_i - x_i} \right)$$

$$= \left[\prod_{i=1}^{m} \binom{n_i}{x_i} \right] \hat{p}_{m,1}{}^{S_m} \left(1 - \hat{p}_{m,1}\right)^{N_m - S_m},$$

$$\hat{p}_{m,1} = \min\left(\hat{p}_m, p_1\right).$$

Probability Ratio

- Probability Ratio for sequence of binomial observations

$$L_m = \frac{f^*_{m,1}}{f^*_{m,0}} = \frac{\hat{p}_{m,1}{}^{S_m}\left(1-\hat{p}_{m,1}\right)^{N_m-S_m}}{\hat{p}_{m,0}{}^{S_m}\left(1-\hat{p}_{m,0}\right)^{N_m-S_m}} = \begin{cases} \left(\dfrac{p_1}{\hat{p}_m}\right)^{S_m}\left(\dfrac{1-p_1}{1-\hat{p}_m}\right)^{N_m-S_m}, & \text{for } \hat{p}_m \le p_0 \\ \left(\dfrac{p_1}{p_0}\right)^{S_m}\left(\dfrac{1-p_1}{1-p_0}\right)^{N_m-S_m}, & \text{for } p_0 < \hat{p}_m < p_1 \\ \left(\dfrac{\hat{p}_m}{p_0}\right)^{S_m}\left(\dfrac{1-\hat{p}_m}{1-p_0}\right)^{N_m-S_m}, & \text{for } p_1 \le \hat{p}_m \end{cases}$$

Sequential Probability Ratio Test

$$Z_m = \ln L_m = \left(C_{1,m} + C_{2,m}\right) S_m - C_{2,m} N_m, \text{ where}$$

$$\mathrm{C}_{1,m} = \ln\left(\frac{r_{1,m}}{r_{0,m}}\right), \mathrm{C}_{2,m} = \ln\left(\frac{1-r_{1,m}}{1-r_{0,m}}\right)$$

- When H_0 true, $Z_m \rightarrow -\infty$
- When H_1 true, $Z_m \rightarrow +\infty$
- Use Wald's decision boundaries $\boldsymbol{a}$ and $\boldsymbol{b}$
  - When $Z_m < \boldsymbol{a}$, reject H_1; accept H_0
  - When $Z_m > \boldsymbol{b}$, reject H_0; accept H_1
  - When $\boldsymbol{a} < Z_m < \boldsymbol{b}$, continue sampling

The Test Statistic

- From previous slide, we see Z_m is
  - a function of the cumulative sum S_m
  - a random walk with
    - Drift $$\mathrm{E}\,Z_m = N_m\left[pC_{1,m} + (1-p)C_{2,m}\right]$$
    - Increasing variance $$\operatorname{var}(Z_m) = N_m p(1-p)\left(C_{1,m} + C_{2,m}\right)$$
  - asymptotically Gaussian, when suitably normalized
  - a martingale for $p = C_2/(C_1 + C_2)$
  - a sub-martingale for $p > C_2/(C_1 + C_2)$
  - a super-martingale for $p < C_2/(C_1 + C_2)$

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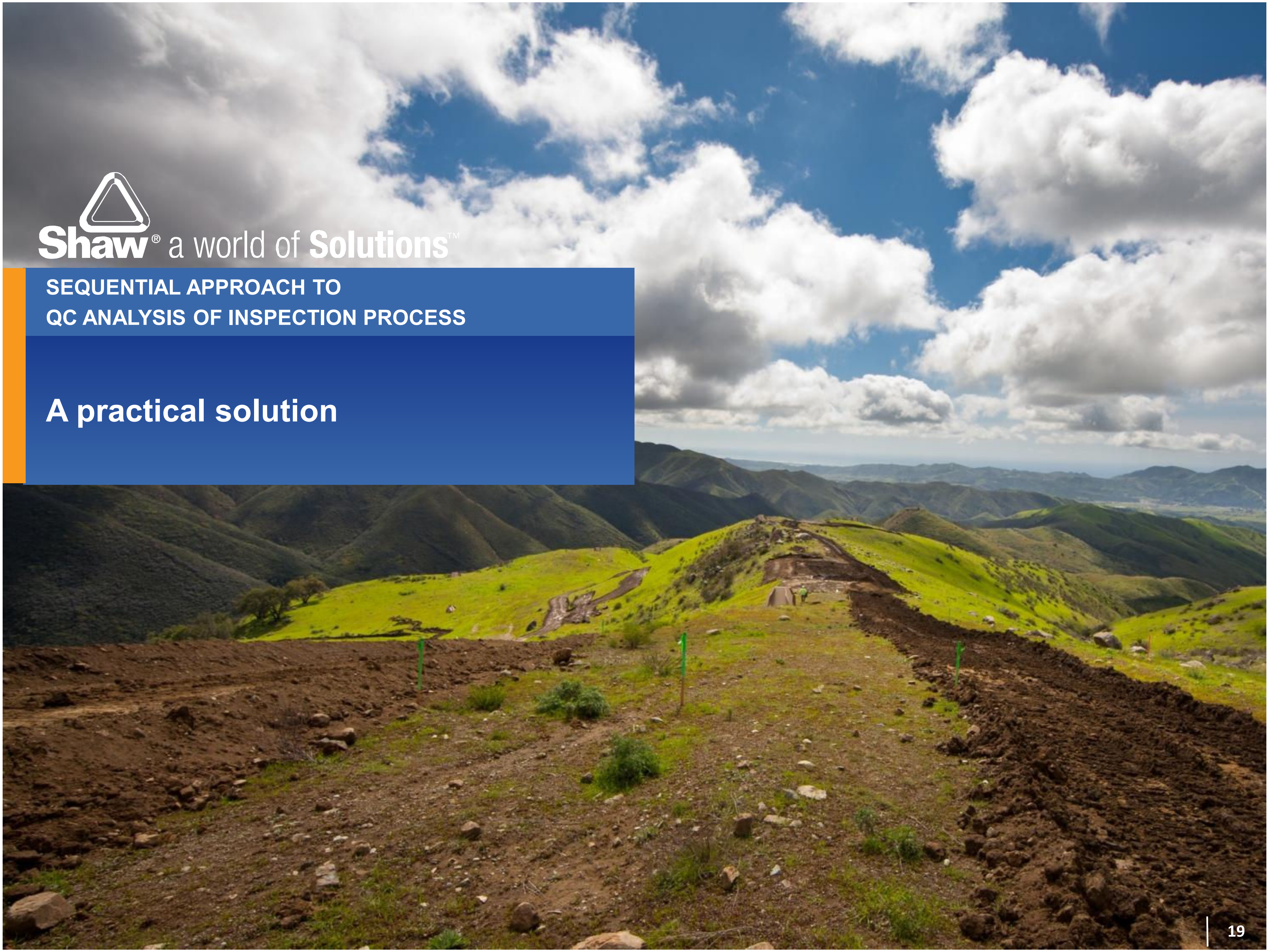
A practical solution



QA Surveillance Strategy

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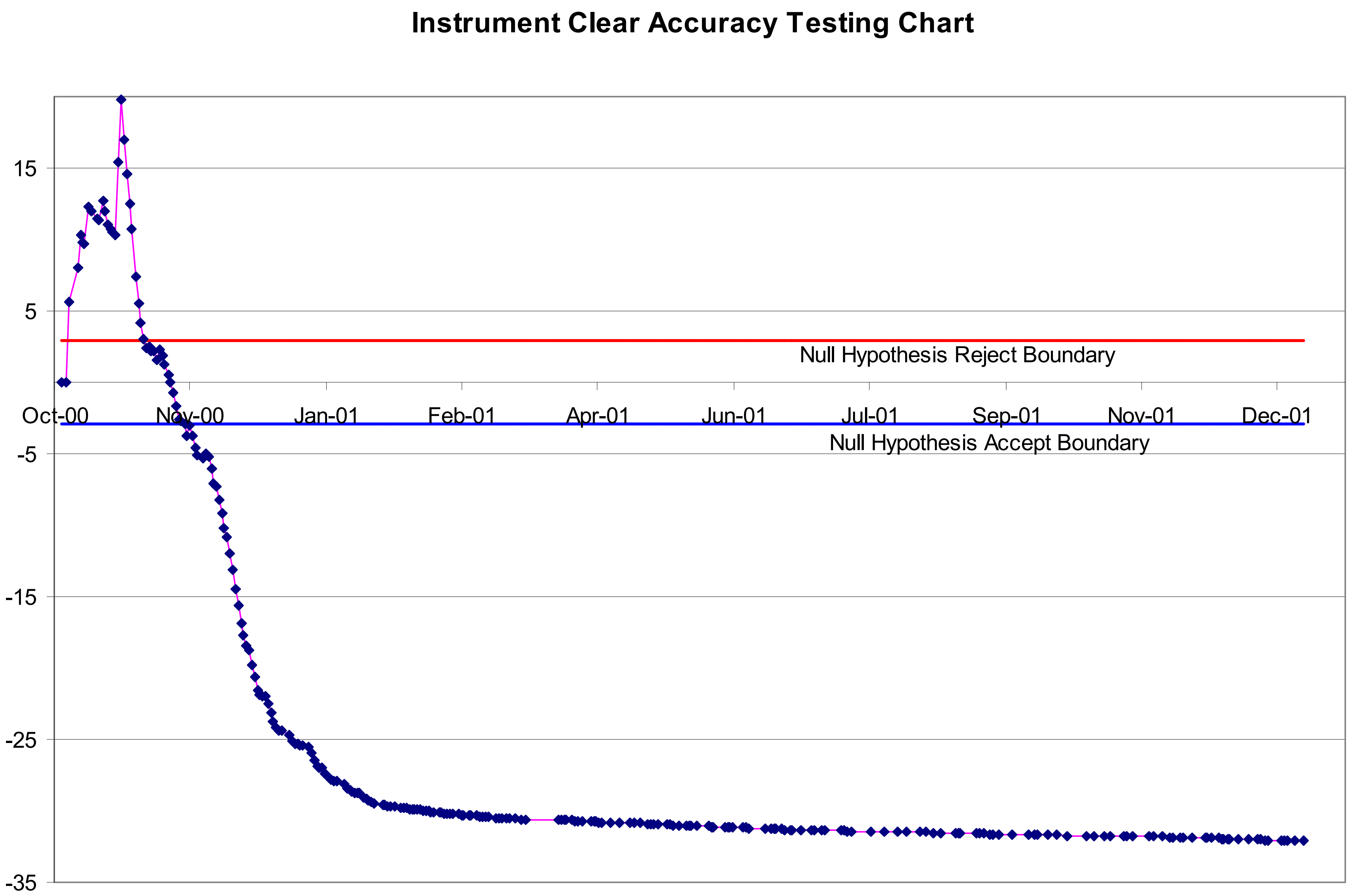
- ►Normal surveillance rate—1 QA check per inspector per day
- ►Enhanced surveillance rate—2 QA checks per inspector per day
- ►Start under normal surveillance
- ►Single detected clearance error puts inspector under enhanced surveillance for the next 5 days
- ►Another detected clearance error renders him ineligible to conduct inspections
- ►All work performed by disqualified inspector must be checked

SPRT Adaptation

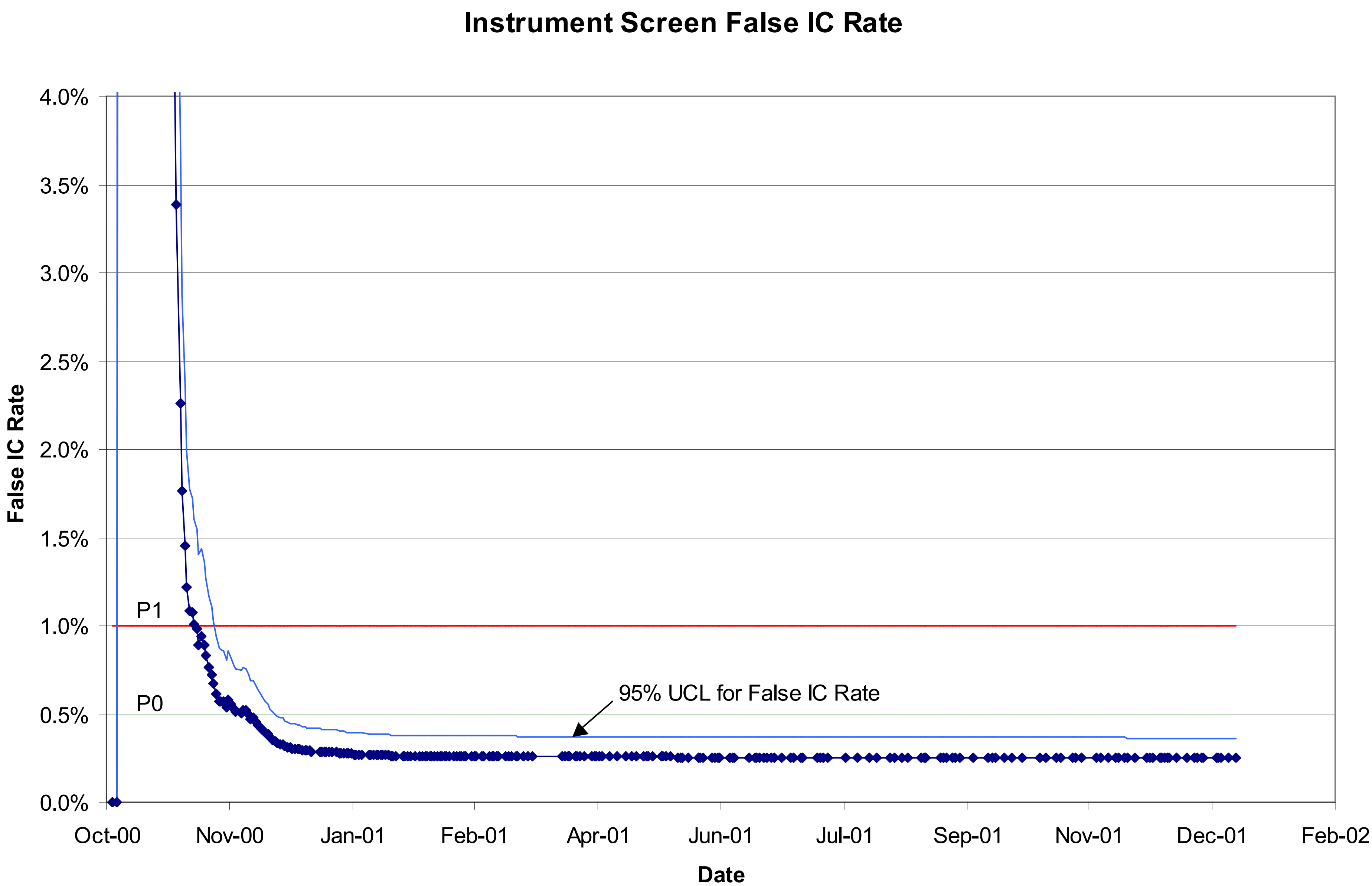
- Test whether false clearance error rates acceptably low
- Test statistic Z_m used as quality indicator
- Observations not terminated until all inspections complete
- Wald's lower boundary a is used but does not terminate sampling
- Test statistic passing a at any time indicates clearance error rates are acceptably low

SPRT Adaptation

- Three consecutive points moving closer to fail decision boundary triggers corrective action investigation
- Test statistic passing fail boundary (***b***) is strong indication of need for corrective action, including
  - rechecking residences,
  - retraining inspectors
  - releasing inspectors
  - firing subcontractor



Instrument Clear Accuracy Testing Chart
15
5
-5
-15
-25
-35
Oct-00
Nov-00
Jan-01
Feb-01
Apr-01
Jun-01
Jul-01
Sep-01
Nov-01
Dec-01
Null Hypothesis Reject Boundary
Null Hypothesis Accept Boundary



Instrument Screen False IC Rate
4.0%
3.5%
3.0%
2.5%
2.0%
1.5%
1.0%
0.5%
0.0%
False IC Rate
P1
P0
95% UCL for False IC Rate
Oct-00
Nov-00
Jan-01
Feb-01
Apr-01
Jun-01
Jul-01
Sep-01
Nov-01
Dec-01
Feb-02
Date

Some Early Stage Statistics (3/1/2001)

Num NI checked	Nn	7,382
Num NI found ID	Xn	0
Rate of NI ID	Est. pn	0.00%
Num Involved checked	Ni	126,292
Num Involved found ID	Xi	1,013
Num Involved found IC	Ni - Xi	125,279
Rate of Involved ID	Est. pi	0.80%

Statistical Quality Support

- Developed statistical methodology
- Won over key regulator
- Developed and installed software for
  - Client/server data entry and management system
  - Statistical analysis
  - Automated reporting (run and emailed nightly)
- Trained project personnel on site
- Operational
  - DBA
  - Software maintenance
  - Monitoring data entry and report generation
- Statistical consulting—error surveys

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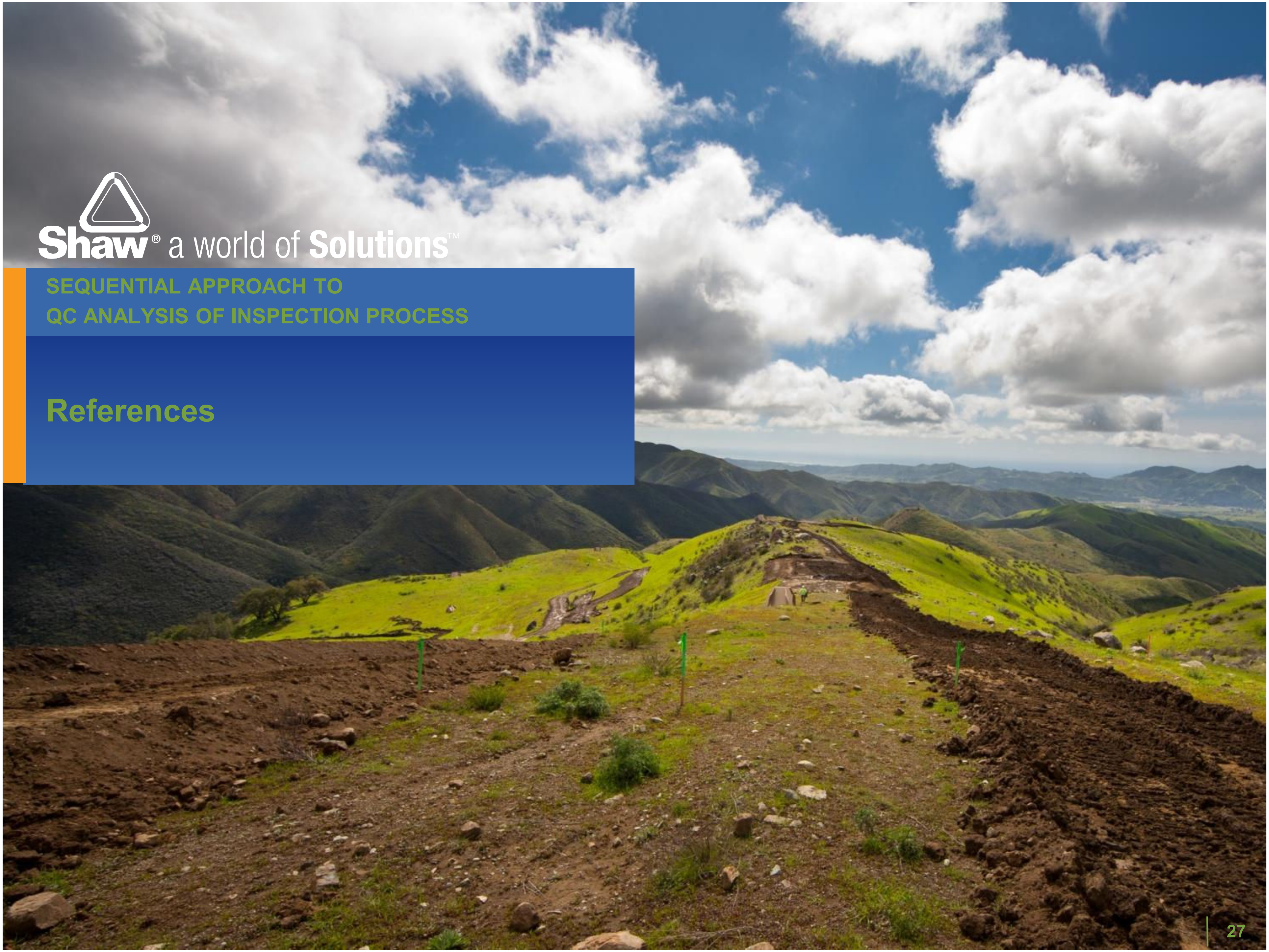
References



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